Column Generation for IMRT Cancer Therapy Optimization with Implementable Segments

by Felisa Preciado-Walters, PhD, Kimberly-Clark Corp

Ronald L. Rardin, PhD, Purdue University

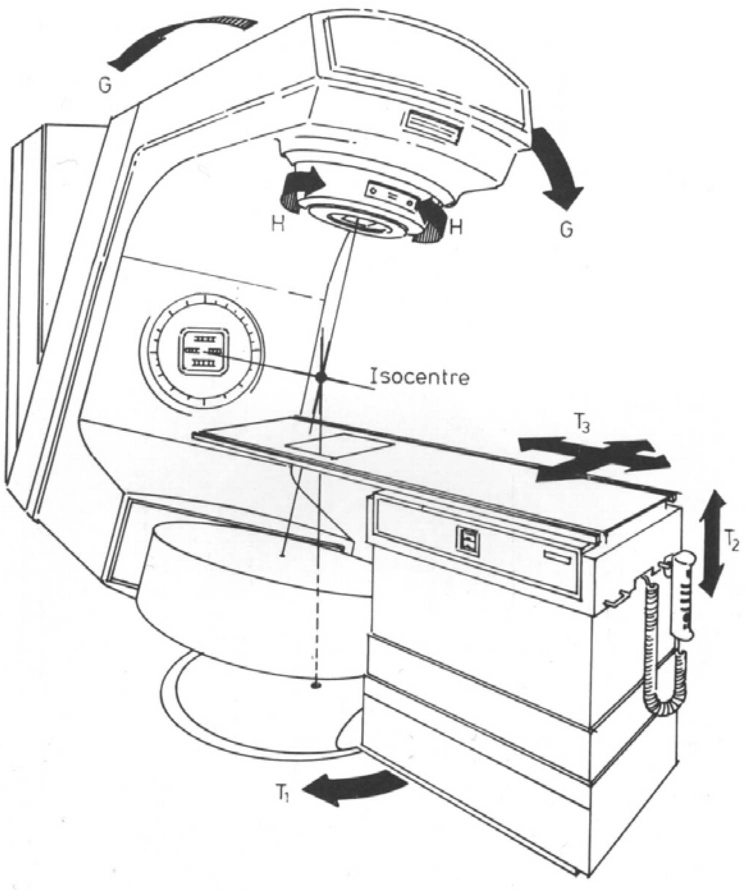
Mark P. Langer, MD and Van Thai, Indiana University School of Medicine

(presented by Delal Dink, Purdue University)

Sponsored in part by NSF grant 0120145 & NCI grant 1R41CA91688-01

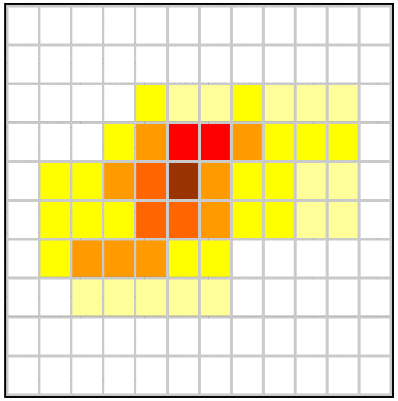
Radiation Therapy Opt

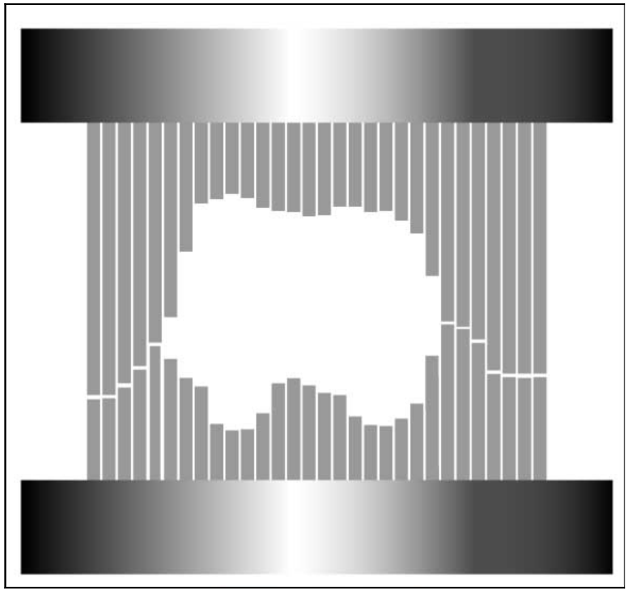
- 1.2M/yr new cases of cancer in U.S. alone
  - Times 10 globally
- Half undergo radiation therapy
- Some treated with implants, but most with external beam



IMRT Refinement

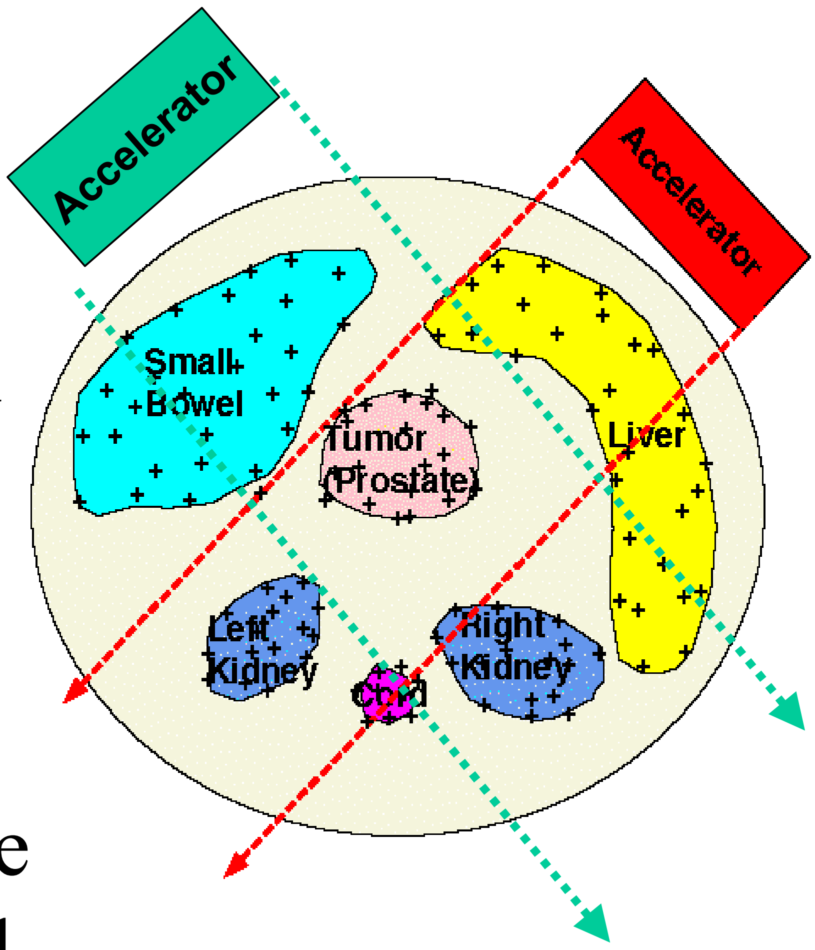
- Beam is big: 10 cm square
- Conformal therapy covers whole image of tumor
- Latest IMRT can deliver shaped array of 3mm beamlets
- Implemented via a Multi-Leaf Collimator (MLC) creating a time-varying opening (leaves can be vertical or horizontal)





Therapy Planning Task

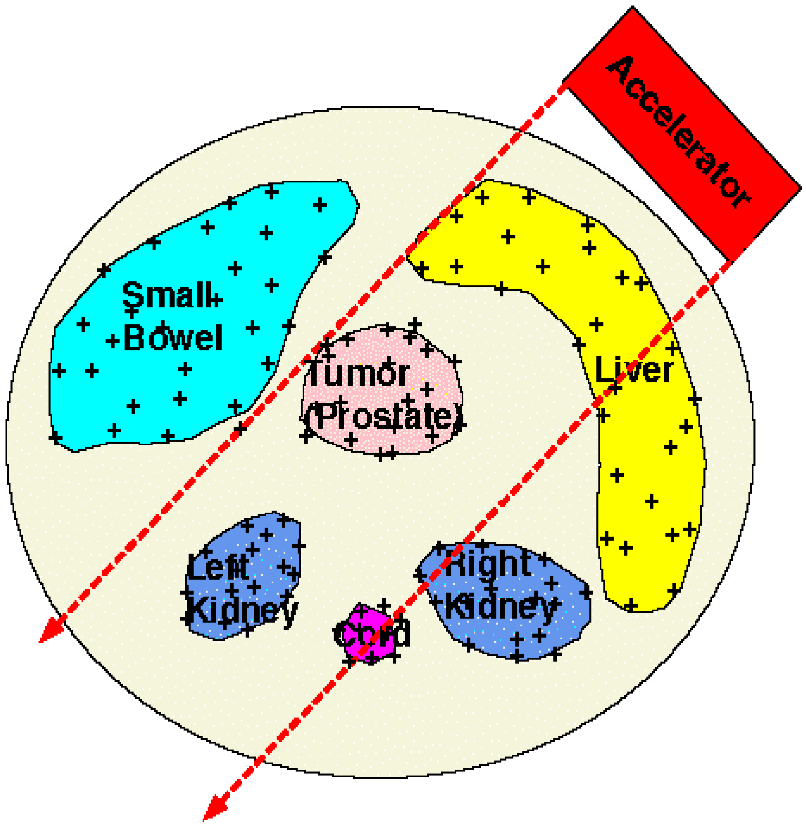
- Choose beam angles, shapes & intensities
- Max tumor dose
- Limit dose to healthy tissues
- Multiple beams spread healthy dose
- 1% higher tumor dose adds 1.5% to survival



Modeling Simplifications

- Shapes: Deal with shapes by discrete points
  - Hundreds per tissue
- Linearity: Dose at any point i linear in beam(let) intensity j

$$\sum a_{ij} x_j$$



Math Program Model

- Objective Function: max minimum dose at any tumor point
- Homogeneity: highest dose tumor point no more than fixed multiple of lowest

Tissue	*No. of points*
Tumor	2385
Bladder	751
Rectum	871
Femoral Head 1	714
Femoral Head 2	720
Sigmoid	1154
Skin	1811

Math Program Model

- Simple dose limits: radiation at any healthy tissue point cannot exceed a specified limit *b*
- Restrict number of beam angles

Tissue	*No. of points*	*b (cGy)*
Tumor	2385	-
Bladder	751	10000
Rectum	871	10000
Femoral Head 1	714	7200
Femoral Head 2	720	7200
Sigmoid	1154	8500
Skin	1811	15000

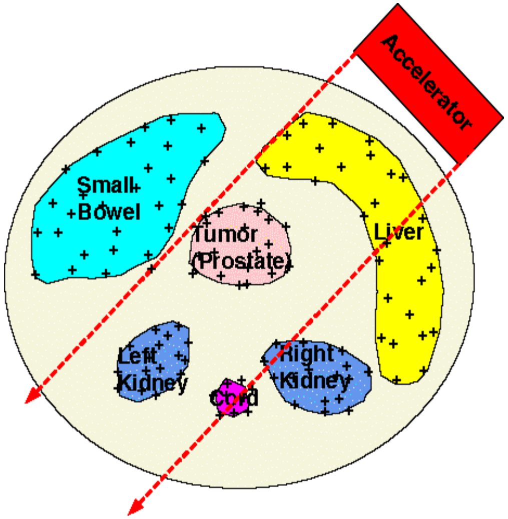
Math Program Model

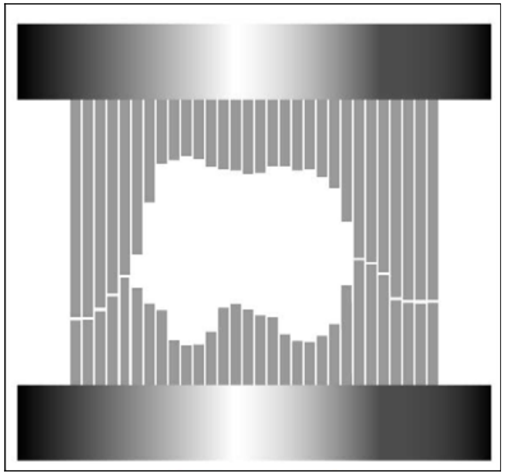
- Dose Volume: constraints restricting the load to $d < b$ on a fraction of a tissue

Tissue	***No. of points***	***b (cGy)***	***% under DV rest.***	***d (cGy)***
Tumor	2385	-	-	-
Bladder	751	10000	80	8000
Rectum	871	10000	80	7500
Femoral Head 1	714	7200	60	5000
Femoral Head 2	720	7200	60	5000
Sigmoid	1154	8500	90	7500
Skin	1811	15000	0	-

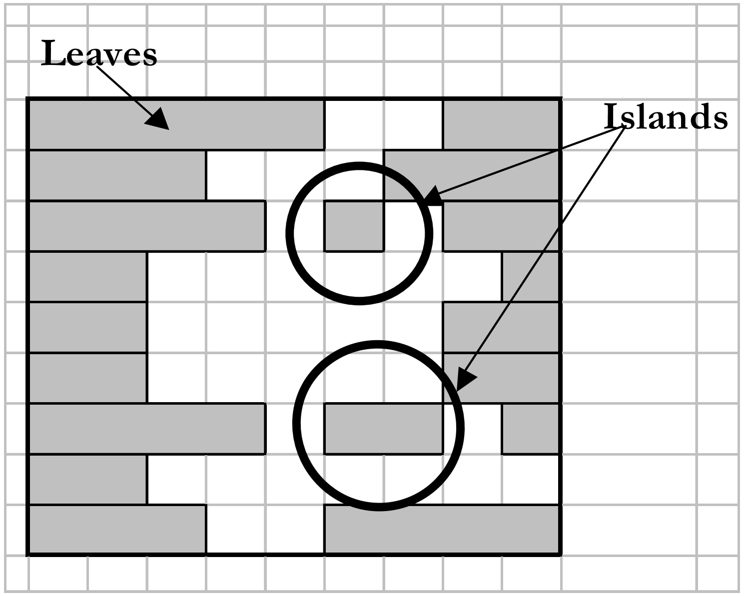
Mixed-Integer Approach

- Model dose-volume and beam-count constraints with binary variables
- Column-generate to find good IMRT segments (open/close)
- Round LP relaxations to produce MIP solns

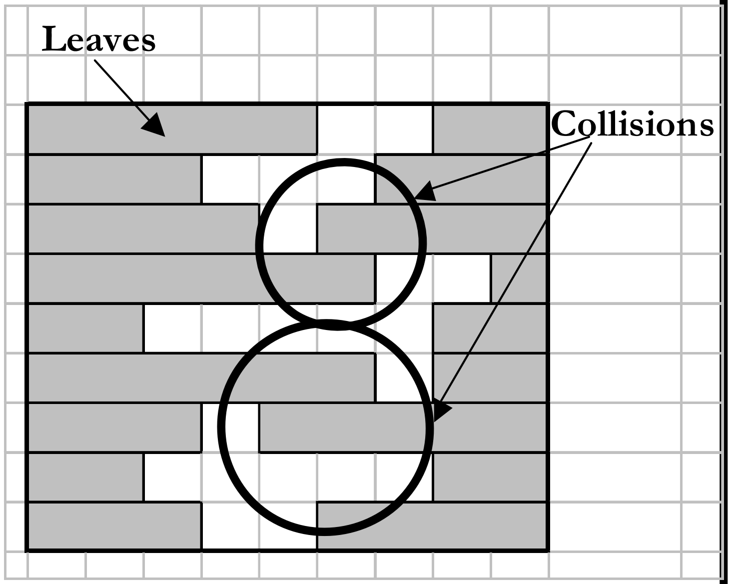




MLC Segment Limitations



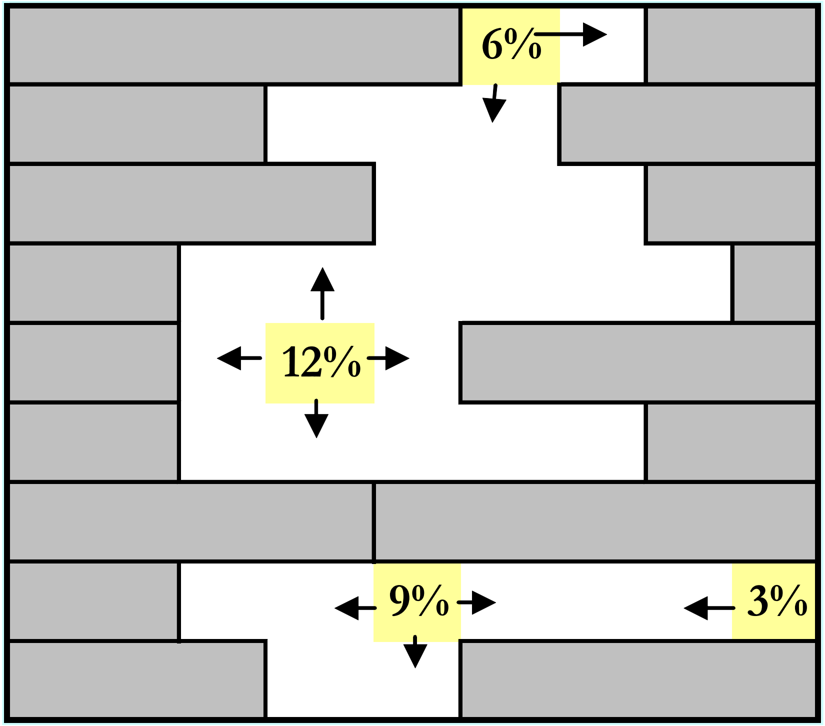
- Islands in rows are not implementable with MLCs



- Collision risk precludes leaf overlap in adjacent rows

MLC Boundary Corrections

- Tongue and Grove design of leaves also introduces boundary effects
- We adjust effective beamlet intensities +3% for every open neighbor



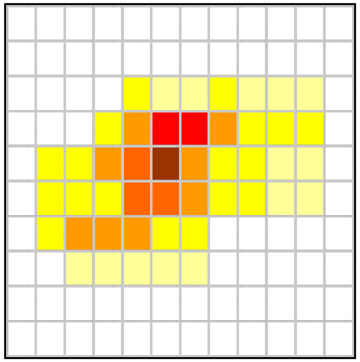
Implementable Segment Column Generation

- Employ classic approach of building column via dual variable values on constraints
  - Sum to a potential = cbar
- Select positives to open, enforcing rules for Island and Collision (*P*-time)
- Apply 3% adjustment and add to LP relaxation

-4	5	-1	-5	-8	-2	-3	0	0	0
0	-4	-1	8	7	0	-1	-9	9	-8
0	-2	6	-1	4	-5	-8	-9	-9	-7
-2	-5	-3	2	3	-7	-5	-7	-8	-5
-5	-3	-7	-4	2	-3	-8	-5	-4	-2
-6	-7	-5	-5	-3	6	9	11	2	0
-7	8	-8	-7	2	10	3	6	-2	-1
-8	-8	-9	5	-9	-4	0	9	-3	-1
-6	-9	-4	-3	-1	-5	-8	-2	-3	0
-5	-8	-8	-7	-4	-6	-2	5	3	0

An Alternative to Two-Stage Approach

- Our MIP column generation approach produces segments and intensities that can be implemented directly
- Most current methods use two stages
  - First, optimize treating beamlets as independent
  - Second, derive a segment plan that approximately delivers optimal beamlet intensities
    - Discretize intensities to 10-15 values
    - Apply segment a generation heuristic
    - Adjust the results for boundary effects



Comparison of Run Times

- In most cases complete (rounded) MIP runs take less CPU than independent beamlets LP alone
- Typically runs more efficiently when MLC segment rules limit column generation

	Independent Beamlets	Colgen without MLC Segment Rules		Colgen with MLC Segment Rules	
Target Site	CPU (mins)	Number Segments	CPU (mins)	Number Segments	CPU (mins)
prostate	452.1	233	29.4	202	19.4
prostate	609.1	137	23.2	146	13.8
prostate	168.8	253	11.3	217	6.8
lung	27.4	822	61.9	560	31.0
naso-pharynx	4.1	318	4.0	256	3.5
thorax	37.5	207	3.5	243	5.9

Delivery Gains (Prostate Case)

- Can measure effects by post-evaluation with many sample pts
- Results (1 case)
  - MIP Direct avoids dangerous infeasibilities
  - MIP Direct requires only half the exposure time
  - MIP Direct uses more segments

	MIP Direct	Two Stage
Min Tumor Dose	99.6 Gy	98.4 Gy
Bladder Limit Violation (over 100Gy)	0.23 Gy	8.59 Gy
Rectum Limit Violation (over 100Gy)	0.15 Gy	8.27 Gy
Skin Limit Violation (over 150Gy)	0.00 Gy	8.67 Gy
Tumor Dose Homogeneity (vs 90%)	0.25%	10.33%
Avg Segs per Angle (9 fixed)	9.0	5.8
Total Fluence	19,723 MU	40,454 MU

Summary

- Column generation offers a flexible approach to dealing with the large numbers of beamlets in IMRT
  - Including permitting enforcement of MLC rules
- Preliminary tests show implied direct implementation often uses less computation than the common two-stage approach and avoids potentially dangerous infeasibilities