PHILOSOPHY 2340
Symbolic Logic
FOURTH (FINAL) EXAMINATION
[Sample]

Part I: Translations

Please translate the following sentences into FOL, using the Tarski's World abbreviations unless otherwise indicated. (5 points each.)

1. There are at most three cubes.

2. The large tetrahedron between a and b is left of a dodecahedron.

3. Both medium tetrahedra are right of c.

4. There are (exactly) three large dodecahedra.

5. There are at most two small cubes to the left of a medium tetrahedron. (This may be ambiguous; translate the reading on which the at most two small cubes are left of the **same** medium tetrahedron.)

6. Neither of the tetrahedra behind a is smaller than the large dodecahedron.

Part II: Proving Systems Equivalent

Consider the following two systems of propositional logic. P is simply the system of propositional rules in our text. P* is different from P in two respects: P* does **not** have the rule of conditional elimination, and instead it **does** have a new rule, the rule that from $P \rightarrow Q$, you can derive $\neg P \vee Q$. Prove that P and P* are equivalent systems. You will need two propositional proofs, plus a brief explanation of why those proofs show that P and P* are equivalent. **Note:** if you find it useful, you may use Taut Con: Excluded Middle to justify any instance of $P \vee \neg P$. If you need excluded middle, this can save you several steps! (5 points for setting up the problem correctly; 10 points for successfully carrying out the required proofs.)

Part III: Alternative Logics

1. Briefly explain the difference between classical logic and intuitionistic logic. (5 points.)

2. Suppose that P has a fuzzy value of .6 and Q has a fuzzy value of .3. Give the fuzzy values of the following (2 points each):

$\neg P$

$\neg(P \vee \neg Q)$

$P \rightarrow Q$

3. Suppose we say that an argument in fuzzy logic is **valid** if and only if, on every interpretation (that is, for every assignment of fuzzy values to atomic sentences), the fuzzy value of the conclusion is greater than or equal to the fuzzy value of the conjunction of the premises. Is the following argument valid, on this definition? Why or why not? (Hint: plug in the values from problem 2 and see what happens.) (4 points)

P

$P \rightarrow Q$

Therefore,

Q

2. Suppose that we have a many-valued logic with three values: true (T), false (F), and neither (N). Construct a three-valued truth table for the connective ‘→’ and justify your choices for the rows involving the value N. (Just use the classical interpretation for the rows that do not involve N.) (10 points.)

Part IV: Proofs

1. Prove that, from the fact that the tetrahedron is large, it follows that it is not the case that there are two tetrahedra. That is, prove that $\neg\exists x\exists y\,(Tet(x) \wedge Tet(y) \wedge x \neq y \wedge \forall z(Tet(z) \rightarrow (z = x \vee z = y))$ from the following premise. It took me 19 steps. (15 points.)

1. $\exists x\,(Tet(x) \wedge \forall y\,(Tet(y) \rightarrow y = x) \wedge Large(x))$

2. This problem involves proving, from the premise that there is exactly one tetrahedron, that there is at most one tetrahedron. Prove $\forall x \forall y\, ((\mathrm{Tet}(x) \land \mathrm{Tet}(y)) \rightarrow x = y)$ from the following premise. This took me 15 steps. (15 points.)

1. $\exists x\, (\mathrm{Tet}(x) \land \forall y\, (\mathrm{Tet}(y) \rightarrow y = x))$