

Errata

1. Page 16. There is an extra $k + 1$ in the last displayed equation.
2. Page 38. The statement of Exercise 4 is incorrect. Rather than letting σ be $(\forall x)(\exists y)[x < y \rightarrow x + 1 = y]$, it should be $(\forall x)(\exists y)[x < y \rightarrow x + 1 \neq y]$. The new σ can be made true or false in a one-element structure, depending upon the definition of $<^{\mathfrak{A}}$.
3. Page 83. Exercise 7. The parenthetical comment should read: (and is a consequence of the Soundness Theorem, Theorem 2.5.3).

4. Page 118. In the statement of Exercise 1, replace $\mathfrak{A} \subseteq \mathfrak{B}$ by $\mathfrak{B} \subseteq \mathfrak{A}$. (Corrected in the Second Printing.)

5. Page 185. There is a flaw in the exposition of the proof of the First Incompleteness Theorem in the case when AXIOM_{OFA} is not represented by a Δ -formula. In this case, the claim that $Deduction_A(\bar{c}, \bar{f})$ is a true Δ -sentence is false, and so Formula (5.2) does not follow.

Rather, to establish (5.2) when AXIOM_{OFA} is not represented by a Δ -formula, we argue as follows: Since AXIOM_{OFA} is recursive, it is represented by a formula $AxiomOfA(e)$. Thus the formula $Deduction_A(c, f)$ clearly *defines* the set DEDUCTION_A. The argument that $Deduction_A$ represents DEDUCTION_A is slightly tangential to the exposition of the text, but (in outline) it goes like this.

Say that $\phi(x)$ is positively numeralwise determined if, for each $a \in \mathbb{N}$, $\mathfrak{N} \models \phi(a)$ implies $N \vdash \phi(\bar{a})$. Say $\phi(x)$ is numeralwise determined if both $\phi(x)$ and $\neg\phi(x)$ are positively numeralwise determined. Thus you can show that ϕ represents Q if and only if ϕ defines Q and ϕ is numeralwise determined. Therefore, to show $Deduction_A$ represents DEDUCTION_A, we must only show that $Deduction_A$ is numeralwise determined.

To do this, we prove an analog to Proposition 4.3.9, showing that atomic formulas are numeralwise determined, and that the collection of numeralwise determined formulas is closed under \neg , \vee , \wedge , and bounded quantification. Since $Deduction_A$ is in the smallest collection of formulas generated by these operations from the atomic formulas (together with $AxiomOfA$), it follows that $Deduction_A$ is numeralwise determined, and thus $Deduction_A$ represents DEDUCTION_A.

We need to establish formula (5.2) on page 185.

Assume that $Thm_A(f)$ is true in the standard model. Then for some natural number c , the formula $Deduction_A(c, f)$ is also true in the standard model. Thus the ordered pair (c, f) is in the set DEDUCTION_A.

Since DEDUCTION_A is represented by the formula $Deduction_A$, this means that the set of axioms N proves the formula $Deduction_A(\bar{c}, \bar{f})$, and then N also proves the formula $Thm_A(\bar{f})$, as needed to establish (5.2).

6. Page 186. The statement and proof of Corollary 5.3.4 are incorrect. In the statement of the Corollary, an additional assumption is needed: Assume, in addition, that A is strong enough to prove all of the axioms of N .

Then in the proof, rather than constructing θ as in Gödel I, we need to have θ be such that

$$N \vdash [\theta \leftrightarrow \neg\gamma(\bar{\ulcorner\theta\urcorner})].$$

From the assumption that θ is provable-from- A , you can show that θ is false-in- \mathfrak{N} . In addition, from the assumption that θ is not provable-from- A , you can prove that θ is true-in- \mathfrak{N} .

Thus θ is either true and unprovable or false and provable.

Then the argument in Gödel I shows that the assumption that θ is false and provable leads to a contradiction, while the argument in 5.3.4 shows that θ can't be true and unprovable. Thus the assumption that Thm_A is recursive leads to a contradiction, as needed.

7. Page 192, line 4. Replace

$$A \vdash \text{Deduction}(\bar{a}, \overline{\rho^1}).$$

with

$$A \vdash \text{Deduction}_A(\bar{a}, \overline{\rho^1}).$$

(Corrected in the Second Printing.)

8. Page 193. Exercise 4. Rather than

$$N \vdash \left[\theta \leftrightarrow \neg \text{Thm}_A(\overline{\neg\theta^1}) \right],$$

it should be

$$N \vdash \left[\theta \leftrightarrow \text{Thm}_A(\overline{\neg\theta^1}) \right].$$

So we really do want θ to assert its own refutability. (Corrected in the Second Printing.)