

**SIGNIFICANT OPEN PROBLEMS IN THE THEORY OF
NON-UNIQUE FACTORIZATIONS**

**AS SELECTED BY THE MAIN SPEAKERS AND
ORGANIZERS OF THE PREP WORKSHOP: THE ART OF
FACTORIZATION IN MULTIPLICATIVE STRUCTURES**

- (1) For which abelian groups G is there a Dedekind HFD R with $\text{Cl}(R) = G$?
- (2) Study $\rho(R)$ for R a Krull domain with $\text{Cl}(R)$ not torsion.
- (3) Study factorization in $R[[x]]$ for R an integral domain (or a Krull domain).
- (4) Which noetherian domains satisfy the main finiteness properties of factorization theory: local tameness, finiteness of the catenary degree and the Structure Theorem for Sets of Lengths ? (cf. Theorem 5.6 in *Non-unique factorizations: a survey*, by A. Geroldinger and F. Halter-Koch).
- (5) If R is a ring of integers of an algebraic number field, then almost all elements of R have catenary degree at most 3 (cf. Theorem 7.4 in *Non-unique factorizations: a survey*).
Which other domains have such a property ? Of course, “almost all” has to be interpreted in a suitable way (i.e., for orders in global fields in the sense of Dirichlet density and for \mathbb{Q} -algebras in the sense of Zariski density).
- (6) Let H be a Krull monoid with finite class group G such that every class contains a prime (the multiplicative monoid of non-zero elements of a ring of integers of an algebraic number field is such a Krull monoid).
Find the precise values of arithmetical invariants of H in terms of the group invariants of G (cf. Chapter 6 in *Non-unique Factorizations. Algebraic, Combinatorial and Analytic Theory*).
- (7) Let M be a Cale monoid (which is not necessarily Krull). Find upper and lower bounds on the elasticity of M . In particular, when is the elasticity finite?

- (8) Study the factorization properties of semigroup rings over Diophantine monoids. In light of the material in Gilmer's book on commutative semigroup rings, what can be said in this special case?
- (9) Study Cale varieties for Diophantine monoids. What can be said in case of just one general equation? What can be said if there are two equations? Determine the associated toric ideal and make a connection to Sturmfels' work in his book "Gröbner bases and convex polytopes."
- (10) If $R[[x]]$ is a UFD, then is $R[[x, y]]$?
- (11) Find an ideal theoretic characterization of "HFD".
- (12) Characterize commutative cancellative monoids M such that $M \cong D^*$ for some integral domain D .
- (13) Let M be a Krull monoid with divisor class group \mathbb{Z} . If $\rho(M) < \infty$, then must $\rho(M)$ be rational?
- (14) Let M be an atomic monoid and $n \in \mathbb{N}$ a positive integer. M is said to be n -factorial if whenever

$$x_1 \cdots x_n = y_1 \cdots y_t$$

where each x_i and y_j is an atom, then $t = n$ and $x_i \simeq y_{\sigma(i)}$ for some permutation σ of $\{1, \dots, n\}$. Does there exist an integral domain D which is n -factorial for some $n > 1$, but not factorial?